TURBULENCE SPECTRUM IN THE RANGE OF HIGH WAVE NUMBERS

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This paper deals with the asymptotic behavior of the spectral function when the wave number increases infinitely. The feasibility of using the results of this analysis in a study of the final time period during which a turbulence degenerates is also discussed.

1. Several reports have dealt with the asymptotic behavior of the turbulence spectrum when $k \to \infty$, where k is the wave number. The behavior of fine-scale eddy perturbations in a velocity field which is linear within distances of the order of the viscosity scale $\eta \sim (\nu^3/\varepsilon)^{1/4}$, where ν is the kinematic viscosity and ε is the mean rate of energy dissipation, has been considered in [1]. The spectrum

$$U(k) \sim \exp(-ck^2) \qquad (c = \text{const}) \tag{1.1}$$

was obtained under the assumption that fine-scale eddies do not interact but receive their energy directly from the perturbations of the order of η.

This approach has been further developed in [2, 3, 4]. The form of the spectrum at $k \to \infty$ was analyzed in [5] with the assumption that direct interactions do play the main role. The results obtained here differ from (1.1):

$$U(k) \sim (\eta k)^2 \exp(-ck) \tag{1.2}$$

Applying the hypotheses about a spectral transfer of energy, one obtains expressions for the spectrum [6, 7] which in some cases agree with either (1.1) or (1.2).

In the range of high wave numbers the turbulence is always almost homogeneous and isotropic and, therefore, for analyzing the spectrum form one may use the "diagrammatic technique" developed by Wyld in [8]. As is well known, the essential difficulty with the turbulence theory arises due to the fact that the system of equations for the moments is not closed. With the diagrammatic technique it becomes possible to express the higher-order moments in terms of three functions: the generalized propagator, the vertex, and the spectral function [8]. As a result, one obtains a system of equations in terms of these functions, but a new difficulty arises in that the right-hand sides of these equations appear as infinite series. The Kraichnan approximation [5] is equivalent to retaining only the first terms of these series on the right-hand sides of the equations [8].

The spectrum within the inertia range differs from that derived by A. N. Kolmogorov [6] and, therefore, it becomes necessary to analyze the effect of subsequent terms in those series.

It will be assumed here that the series are convergent in the range of high wave numbers and that, therefore, one need consider only an arbitrarily large but finite number of series terms.

2. For simplicity, as was done in [8], we will start with the model equation:

$$(-i\omega + \nu k^2)\, v(\mathbf{k}, \omega) = f(\mathbf{k}, \omega) + g \int v(\mathbf{q}, \alpha)\, v(\mathbf{k} - q, \omega - \alpha)\, d\mathbf{q}\, d\alpha \tag{2.1}$$

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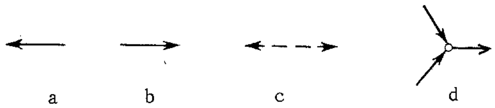

Fig. 1

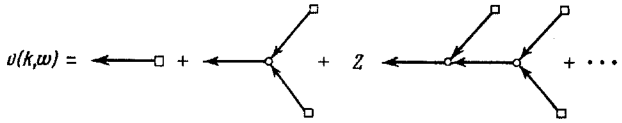

Fig. 2

where $f(\mathbf{k}, \omega)$ is an external force and g is the interaction constant. It will be shown in Sec. 5 that all subsequent calculations remain valid also if one starts with the Navier-Stokes equation. As a convenience for the analysis which follows, we will somewhat modify the graphic symbols used in [8].

The propagator $S(\mathbf{k}, \omega)$ will be referred to by an arrow (Fig. 1a). The quantity $S(-\mathbf{k}, -\omega)$ will be referred to by an arrow pointing backward (Fig. 1b). We now consider the quantity

$$S(\mathbf{k}, \omega) \langle |f(\mathbf{k}, \omega)|^2 \rangle S(-\mathbf{k}, -\omega)$$

The symbol < > signifies averaging over the entire set. This quantity will be referred to by a dashed line. In accordance with what has been said so far, one must assign opposite directions to the endpoints of this line (Fig. 1c). The vertex designations will be the same as in [8]. The external force $f(\mathbf{k}, \omega)$ will be represented by a small square.

Function $v(\mathbf{k}, \omega)$ can be expressed as a series expansion in terms of the perturbation force [8].

This is shown by graphic symbols in Fig. 2. One may analogously rewrite other series representing the generalized propagator, which will here be referred to by a heavy arrow (Fig. 3), the spectral function, and the vertices.

It is easy to see that the spectral function can be defined as the sum of all possible graphs constructed from the elements of Fig. 1 and having two exits.

We will now consider the highest-order moment

$$\langle v(\mathbf{k}_1\omega_1)\, v(\mathbf{k}_2\omega_2) \ldots v(\mathbf{k}_n\omega_n) \rangle$$

In order to represent it graphically, one must express here $v(\mathbf{k}_i, \omega_i)$ $(i = 1, 2, \ldots, n)$ in the form of Fig. 2. As a result, one obtains then the sum of all possible graphs comprising the elements of Fig. 1 and having exits. The n-th order moment will be represented graphically by a shaded circle with n exits (Fig. 4a). The second-order moment, i.e., the spectral function, will be referred to by a heavy dashed line (Fig. 4b). By virtue of the homogeneity and steady-state conditions of turbulence, the impulses and the frequencies of the external lines are respectively constrained by one condition,

$$\mathbf{k}_1 + \mathbf{k}_2 + \ldots + \mathbf{k}_n = 0, \qquad \omega_1 + \omega_2 + \ldots \omega_n = 0 \tag{2.2}$$

We will variationally differentiate the series of Fig. 2 term-by-term with respect to the external force and then average the result (for a definition of the variational derivatives see, e.g., [6]). We obtain thus the sum of all possible graphs comprising the elements of Fig. 1 with one entrance and one exit. It is easy to see that this is how to determine the generalized propagator $S'(\mathbf{k}, \omega)$. Function $S'(\mathbf{k}, \omega)$ describes the response to an infinitesimal perturbation in the external force.

If $F(\mathbf{k}, \omega)$ denotes an infinitesimal nonrandom increment to the external force field, then the averaged response $u(\mathbf{k}, \omega)$ can be expressed in the linear approximation as

$$u(\mathbf{k}, \omega) = S'(\mathbf{k}, \omega)\; F(\mathbf{k}, \omega) \tag{2.3}$$

Kraichnan in [5] arrived at the same result from different considerations.

One can state more generally: to the sum of all possible graphs with m entrances and n exits there corresponds the m-ple variational derivative of the n-th point moment of the velocity field. Such a quantity will be represented graphically by a shaded rectangle with m entrance lines and n exit lines (Fig. 4b). At every vertex there converge three lines: two of them enter and one exits while the impulse and the frequency are maintained. The sum of impulses of the entering lines is equal to the impulse of the exiting line. The same applies to the frequencies. From this follows the relation for any graph with m entering lines and n exiting lines.

If $\underline{K_i}$, ω_i are the impulses and the frequencies of exiting lines and $\underline{p_j}$, α_j are those of entering lines, then

$$\mathbf{k}_1 + \mathbf{k}_2 + \ldots + \mathbf{k}_n = \mathbf{p}_1 + \mathbf{p}_2 + \ldots + \mathbf{p}_m \tag{2.4}$$

$$\omega_1 + \omega_2 + \ldots + \omega_n = \alpha_1 + \alpha_2 + \ldots + \alpha_m \tag{2.5}$$

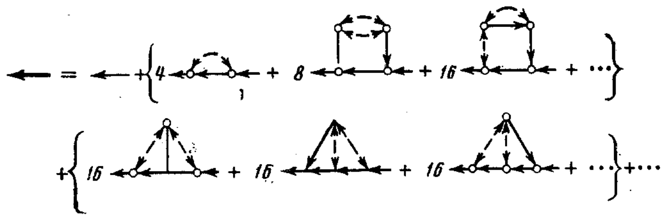

Fig. 3

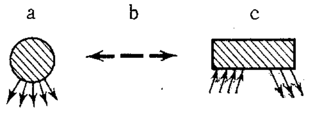

Fig. 4

If the graph is connected, then its external impulses and frequencies are respectively constrained by only one condition of the (2.4), (2.5) kind.

This property helps to prove the statement that the impulses and the frequencies of some number of lines are interrelated if and only if upon cutting these lines the graph will split into two or more parts.

Indeed, the conservation of impulse and frequency at every vertex become the only restriction on the impulses and the frequencies of the internal lines. Therefore, the relation of the (2.4), (2.5) kind is the only possible one. We will assume that the impulses and the frequencies of a certain number of lines are related as per (2.4), (2.5) but that the graph does not fall apart when these lines are cut. One can then still pick some number of lines so that the graph should split into two connected parts. But now two conditions of the (2.4), (2.5) kind can be imposed on both the impulses and the frequencies of each part and, consequently, we have arrived at a contradiction.

3. We will now analyze the relation in Fig. 3. In each graph on the right-hand side there is a solid line passing through the entire graph. We first sum up all the graphs in which this line is connected to the remaining part of the graph through two lines, then we sum up those where there are three connecting lines, etc. The result of this summation is shown in Fig. 5.

To the shaded circle with exits there corresponds a point moment of the velocity field. If it is designated by $U(q_1, \alpha_1, q_2, \alpha_2, \ldots, q_{n-1}, \alpha_{n-1})$, then the relation in Fig. 5 can be rewritten in the analytical form

$$S'(\mathbf{k},\omega) = S(\mathbf{k},\omega) + S(\mathbf{k},\omega)\sum_{n=2}^{\infty}(2g)^n\int U(\mathbf{q}_1\alpha_1\ldots\mathbf{q}_{n-1}\alpha_{n-1})\,S(\mathbf{k}+\mathbf{q}_1,\omega+\alpha_1)$$
$$\times S(\mathbf{k}+\mathbf{q}_1+\mathbf{q}_2,\omega+\alpha_1+\alpha_2)\;S(\mathbf{k}+\mathbf{q}_1+\ldots\mathbf{q}_{n-1},\;\omega+\alpha_1+\ldots+\alpha_{n-1})$$
$$d\mathbf{q}_1 d\alpha_1\ldots d\mathbf{q}_{n-1}d\alpha_{n-1} \tag{3.1}$$

Let us consider the asymptotic behavior of $S'(\underline{k},\omega)$ at $k\to\infty$. We denote the cutoff impulse by k° and the cutoff frequency by ω°, beyond which the spectral function $U(k,\omega)$ decreases fast (e.g., exponentially). If the Reynolds number is sufficiently high, then k° and ω° can be expressed in the viscosity scale by

$$k^\circ \sim \eta^{-1}, \qquad \omega^\circ \sim \nu\eta^{-2}$$

It is further assumed that all functions $U(\mathbf{q}_1\alpha_1\ldots\mathbf{q}_{n-1}\alpha_{n-1})$ are outside the region $|q_i|\lesssim k^\circ$, $|\alpha_i|\lesssim\omega^\circ$ $(i=1,2,\ldots,n-1)$ so that integration can be performed only the region where the integration variables are bounded.

In this region the integrand functions

$$S(\mathbf{k}+\mathbf{q}_1+\ldots+\mathbf{q}_S,\omega+\alpha_1+\ldots+\alpha_S) \qquad (S=1,2,\ldots,n-1)$$

may be expanded with respect to the small parameters q_i/k, $\alpha_i/\nu k^2$ and one need consider only the first term $S(\mathbf{k},\omega)$.

After that, all $S(\mathbf{k},\omega)$ factors can be taken out in front of the integral sign and integration may be extended to infinity.

Using the inverse Fourier transform, it is easy to show that the integrals are proportional to the quantities $\langle[v(\mathbf{x},t)]^n\rangle$, i.e., to the single-point moments in the space-time representation.

In the case of homogeneous and isotropic turbulence, the values of $\langle[v(\mathbf{x},t)]^n\rangle$ will be bounded by constants. Relation (3.1) will then become

$$S'(\mathbf{k},\omega) = S(\mathbf{k},\omega)\{1+\Sigma[2(2\pi)^4 gS(\mathbf{k},\omega)]^n\langle[v(\mathbf{x},t)]^n\rangle\} \tag{3.2}$$

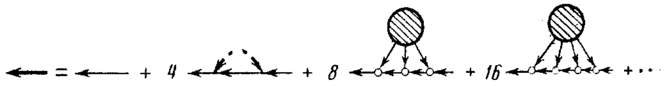

Fig. 5

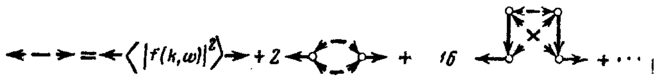

Fig. 6

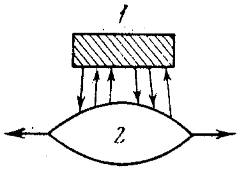

Fig. 7

As $k \to \infty$, the propagator $S(\mathbf{k}, \omega) \sim 1/\nu k^2$ and, therefore,

$$S'(\mathbf{k}, \omega) \approx S(\mathbf{k}, \omega) \tag{3.3}$$

When $\omega \to \infty$, then analogous calculations also lead to the relation (3.2) in the form of an expansion with respect to the small parameter $\sim 1/\omega$ and we again have the equality (3.3).

We will explain the physical meaning of expression (3.3). Using the Dayson equation [8]

$$S'(\mathbf{k}, \omega) = S(\mathbf{k}, \omega) + S(\mathbf{k}, \omega)\,\Sigma_1(\mathbf{k}, \omega)\,S'(\mathbf{k}, \omega) \tag{3.4}$$

(the function $\Sigma_1(\mathbf{k}, \omega)$ is defined in [8]), we can rewrite relation (2.3) as follows:

$$(-i\omega + \nu k^2)\; u\;(\underline{\mathbf{k}}, \omega) = \Sigma_1(\mathbf{k}, \omega)\, u(\mathbf{k}, \omega) + F(\mathbf{k}, \omega) \tag{3.5}$$

Relation (3.5) has the form of a linearized Eq. (2.1) with the additional term $\Sigma_1(\mathbf{k}, \omega)\,\mathbf{u}(\mathbf{k}, \omega)$ which describes the effect of turbulent viscosity. Disregarding the latter will again, according to (3.4), lead to the equality (3.3). In this way, turbulent pulsations have no effect on the buildup of perturbations with wave numbers $k \gg k^\circ$ or at frequencies $\omega \gg \omega^\circ$.

We will next analyze the behavior of the vertex function [8]:

$$\Gamma(\mathbf{k}, \omega, \mathbf{k}', \omega')$$

when the impulses or the frequencies of any external lines are large.

If both the impulses or the frequencies of all external lines are large, then by reasoning as before we will arrive at the following relation:

$$\Gamma(\mathbf{k}, \omega, \mathbf{k}', \omega') \approx g \tag{3.6}$$

If both the impulse and the frequency of any entering line, such as k' and ω', are small but not equal to zero, however, then (3.6) remains valid if the additional condition that the variational derivatives be finite

$$\delta \langle [v(\mathbf{x}, t)]^n \rangle / \delta f(\mathbf{k}', \omega')\, d\mathbf{k}' d\omega'$$

is also stipulated.

The finiteness of variational derivatives means simply that the spectrum is stable, within the linear approximation, with respect to perturbations having the wave number k' and the frequency ω'.

4. The equation for the spectral function $U(k, \omega)$ [8] is shown in Fig. 6. We will assume that the series on the right-hand side of Fig. 6 is convergent so that for analyzing the asymptotic behavior of the spectral function one need consider a finite number of terms only.

The complexity of the graphs is based on the following rule: as one dashed line is added, one integration as well as two vertices and two solid lines are also added. The number of dashed lines always exceeds the number of integrations by one and, therefore, a relation must necessarily exist between the impulses and between the frequencies of some number of dashed lines.

In order to explain this, we note that it is possible by a single cut to split each graph, except the first one, on the right-hand side of Eq. (Fig. 6) into two parts with a break in the dashed lines only. The external lines of such a graph will then fall into different parts. All dashed lines which are thus cut will be grouped into Class A and all the other dashed lines into Class B. By virtue of (2.4), (2.5), the impulses and the frequencies of Class A lines are respectively constrained by one condition:

$$q_1 + q_2 + \ldots + q_l = k, \qquad \beta_1 + \beta_2 + \ldots + \beta_l = \omega \tag{4.1}$$

Here l is the number of lines in Class A, q_i and β_i are their impulses and frequencies, **k** and ω are the impulse and the frequency of the spectral function on the left-hand side of the equation. In accordance with # 2, the impulses and the frequencies of Class B lines are independent. It follows, then, that the impulses and the frequencies of the dashed lines may serve as integration variables, but with the function

$$\delta\left(\sum_{i=1}^{l} \mathbf{q}_i - \mathbf{k}\right)\delta\left(\sum_{i=1}^{l} \beta_i - \omega\right) \tag{4.2}$$

included in the integrand.

We will now analyze the case $k \to \infty$. In the integration with respect to impulses and frequencies of Class B lines one need consider only the region where the variables are bounded. In the integration with respect to $\mathbf{q}_i$, β_i $(i = 1, 2, \ldots, l)$, however, this is not permissible with function (4.2) under the integral sign.

Let us consider a region of integration where a graph can be split into two parts connected through lines with small impulses and frequencies. One can always split the graph into two parts in such a way that part 2 will not contain lines whose impulses are simultaneously small. We now add up all the graphs with the same such part 2, with the same number and direction of connecting lines, but with a different part 1 structure. The result of this summation is shown in Fig. 7.

In accordance with the conclusions of the preceding paragraph, the generalized vertices and response functions of part 2 may be replaced by nuclear vertices according to formulas (3.3) and (3.6). The functions contained in part 2 can be expanded into series with respect to small impulses and frequencies of the connecting lines and one need consider the first term only. After that, part 2 ceases to depend on the impulses and the frequencies of the connecting lines, the integration with respect to variables on which parts 1 and 2 depend can be performed separately.

Let us designate by $R(\mathbf{k}_1, \omega_1, \ldots, k_n, \omega_n, p_1, \alpha_1, \ldots, p_m, \alpha_m)$ the function which corresponds to part 1 (Fig. 7). The meaning of the arguments is the same as in (2.4), (2.5). In accordance with Sec. 2,

$$\delta^m \langle v(\mathbf{k}_1\omega_1)\ldots v(\mathbf{k}_n\omega_n)\rangle = R(\mathbf{k}_1\omega_1\ldots \mathbf{k}_n\omega_n; \mathbf{p}_1\alpha_1\ldots \mathbf{p}_m\alpha_m)\,\delta f(\mathbf{p}_1\alpha_1)\ldots\delta f(\mathbf{p}_m\alpha_m) \tag{4.3}$$

It will be assumed that the integral of function R over all arguments within a limited range is finite. According to (4.3), this assumption is equivalent to the requirement that the quantities $\langle [v(\mathbf{x}, t)]^n\rangle$ be stable with respect to infinitesimal variations of the external force

$$\delta f(\mathbf{p}, \alpha) = \varkappa \quad \text{for} \quad |\mathbf{p}| < p_0, \ |\alpha| < \alpha_0$$
$$\delta f(\mathbf{p}, \alpha) = 0 \quad \text{for} \quad |\mathbf{p}| > p_0 \quad \text{or} \quad |\alpha| > \alpha_0$$

Here $\varkappa$ is some small constant and p_0, α_0 are certain impulses and frequencies which satisfy the conditions $p_0 \ll k$, $\alpha_0 \ll vk^2$.

When $m = 0$, the requirement that the integral of the function in part 1 be bounded corresponds to the condition that the quantities $\langle [v(\mathbf{x}, t)]^n\rangle$ be bounded.

It is easy to see that in every case an increase in the number of connecting lines results in an increased number of lines with a large impulse or frequency in part 2 and, therefore, the analyzed graph will be smaller than some lower-order graph. Consequently, on the right-hand side of the equation one must retain only those graphs which do not contain Class B lines and in the integrations one has to consider only the region where the impulses or the frequencies of all lines are large.

We will express function $U(k, \omega)$ as

$$U(k, \omega) = U(k)\, r(k, \omega) \tag{4.4}$$

where $r(k, \omega)$ is normalized by the condition

$$\int r(k, \omega)\, d\omega = 1 \tag{4.5}$$

Let us find a solution for $U(k)$ in the form

$$U(k) = \psi(k) \exp\left[-a(k/k^\circ)^\gamma\right] \tag{4.6}$$

Here a, γ are certain constants, ψ (k) is a function which varies with k not faster than as a power. Function r (k, ω) signifies the frequency distribution of energy in a harmonic with the wave number k.

Let ω_*(k) denote the characteristic frequency above which r (k, ω) decreases fast. One could reason that ω_*(k) increases with k not faster than k^2. To the first approximation, which is the concern of this analysis, one only has to determine the exponent γ.

For this purpose, it suffices to assume that ω_*(k) increases with k not faster than as some power.

We substitute (4.4), (4.6) into the equation for U(k, ω) and then integrate its both sides with respect to ω. The second δ-function in (4.2) will then vanish and any graph on the right-hand side of Eq. (Fig. 6) containing l Class A lines will be written as

$$\int \exp\left(-a\sum_{i=1}^{l} q_i^{\gamma}\right)\delta\left(\sum_{i=1}^{l}\mathbf{q}_i-\mathbf{k}\right)d\mathbf{q}_1\ldots d\mathbf{q}_l \int\limits_{|\beta_1|=0}^{\beta_1=\omega_*(q_1)}\ldots\int\limits_{|\beta_l|=0}^{\beta_l=\omega_*(q_l)}\Psi(q_1\beta_1,\ldots \ldots, q_l\beta_l)\,d\beta_1\ldots d\beta_l \tag{4.7}$$

Here Ψ includes the functions S, r (q, β), and the vertices. As $q_1,\ldots,q_l$ changes, the integral with respect to $\beta_1,\ldots,\beta_l$ varies not faster than as a power and, therefore, the region where the exponential factor is maximum, i.e., where

$$\sum_{i=1}^{l} g_i^{\gamma}\delta\left(\sum_{j=1}^{l}\mathbf{q}_j-\mathbf{k}\right)$$

is minimum adds the main contribution to the integral with respect to $q_1,\ldots,q_l$. It is easy to verify that this minimum occurs at $q_1=q_2=\ldots=q_1=k/1$ when $\gamma\neq 1$. When $\gamma=1$, on the other hand, this minimum occurs at parallel $q_1,\ldots,q_l$.

We will now assume the spectrum of external forces $\langle|f(k,\omega)|^2\rangle$ to be upper bounded so that within the range of large k one may disregard the first term on the right-hand side of Eq. (Fig. 6). By virtue of what has been said, Eq. (Fig. 6) for $\gamma\neq 1$ can be rewritten as

$$\exp\left[-a\left(\frac{k}{k^{\circ}}\right)^{\gamma}\right]=\sum_{l=2}^{N}\varphi_l(k)\exp\left[-al\left(\frac{k}{k^{\circ}l}\right)^{\gamma}\right] \tag{4.8}$$

where φ_1(k) are certain functions which vary with k not faster than as a power and N is an arbitrarily large but finite integer. Multiplying both sides of (4.7) by $\exp[a(k/k^{\circ})^{\gamma}]$ yields

$$1=\sum_{l=2}^{N}\varphi_l(k)\exp\left[-a\left(\frac{k}{k^{0}}\right)^{\gamma}\left(\frac{1}{l^{\gamma-1}}-1\right)\right] \tag{4.9}$$

It is easy to see that this equation cannot be satisfied when $k\to\infty$, if l differs from unity. Therefore,

$$U(k)\sim\exp[-ak/k^{\circ}] \tag{4.10}$$

5. The entire reasoning remains valid when Eq. (2.1) is replaced by the Navier-Stokes equation. In the spectral form this equation is [8]:

$$(-i\omega+\nu k^2)v_i(\mathbf{k},\omega)=f_i(\mathbf{k},\omega)+P_{ijl}(\mathbf{k})\int v_j(\mathbf{q},\alpha)v_l(\mathbf{k}-\mathbf{q},\omega-\alpha)\,d\mathbf{q}d\alpha$$

$$P_{ijl}(\mathbf{k})=-\frac{i}{(2\pi)^4}[k_j\Delta_{il}(\mathbf{k})+k_l\Delta_{ij}(\mathbf{k})],\ \Delta_{ij}(\mathbf{k})=\delta_{ij}-\frac{k_ik_j}{k^2}$$

In the case of a homogeneous isotropic turbulence the spectral tensor is defined by one scalar function [6]:

$$U_{ij}(k,\omega)=\Delta_{ij}(\mathbf{k})\,U(k,\omega)$$

Function U(k, ω) can be expressed in the form (4.4), (4.6). Since $P_{ijl}(\mathbf{k})\sim k$, hence

$$kS(k,\omega)=\frac{k}{-i\omega+\nu k^2}$$

will serve as the parameter with respect to which the disregarded graphs are indeed small when either $k\to\infty$ or $\omega\to\infty$.

The integrand in (4.7) will contain a factor which depends on the angle between vectors $q_1, \ldots, q_l$. but its maximum will – as before – occur at $q_1 = q_2 = \ldots = q_l = k/l$ within the accuracy of the power exponent.

The validity of (4.10) can be verified by a comparison with experimental data on turbulence in a pipe. There is, however, another area where it can be useful: in analyzing the final time period when turbulence degenerates. One assumes here usually that

$$\frac{\partial U(\mathbf{k}, t)}{\partial t} \approx 2vk^2 U(\mathbf{k}, t) \tag{5.1}$$

with nonlinear interactions completely disregarded. The solution to (5.1) is

$$U(\mathbf{k}, t) = C \exp\left(-2vk^2 (t - t_0)\right]$$

At the beginning of the final period during which turbulence degenerates one may justify disregarding the nonlinear terms by the fact that the rate at which pulsations are viscously damped will be much higher than the rate at which energy is transmitted nonlinearly across the spectrum. With time, however, the nonlinear interactions will become the only energy source for the harmonics with a sufficiently high wave number and, therefore, they may not be disregarded. Formula (4.10) begins to become valid now, with the largest-scale perturbations acting as the slow time-varying factor in front of the exponential term.

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